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RICHTMYER–MESHKOV INSTABILITY OF AN INTERFACE BETWEEN TWO MEDIA DUE TO PASSAGE OF TWO SUCCESSIVE SHOCKS

A. A. Charakhch'an

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The instability of a free surface of aluminum after passage of two shocks that follow one after the other at a certain time interval is studied numerically. The first shock is rather strong (the postshock pressure is about 75 GPa*). It is shown that if at the moment when the second shock arrives at the free surface, the perturbation evolution is nonlinear, then, in contrast to the linear stage, the change in the growth rate of the amplitude depends weakly on the wavelength of the initial perturbation. A formula is proposed which describes the effect of the second shock on the amplitude growth rate and in which the main structure of Richtmyer's formula is preserved. It is demonstrated that the parameters of the second shock that ensure freezing of the instability can be determined using only the growth rate of the amplitude.*

Introduction. The instability of an interface between two media due to passage of a shock wave, which is called Richtmyer–Meshkov instability [1, 2], has been studied in many papers (see, e.g., [3–8] and the bibliography in [8]). In particular, the interest in Richtmyer–Meshkov instability is motivated by continuing research in the field of controlled inertial fusion.

Let us describe the basic features of development of this instability. Let a shock propagate from a medium with density ρ_1 into a medium with density ρ_2. The case of passage of the shock from a light material into a heavy material ($\rho_1 < \rho_2$) and the opposite situation ($\rho_1 > \rho_2$) are distinguished. In both cases, the shock first leads to a certain decrease in the perturbation amplitude, and then the amplitude grows linearly with time. The behavior of the perturbation phase is different. In the case $\rho_1 < \rho_2$, the perturbation phase does not change, whereas in the case $\rho_1 > \rho_2$, the growth in the amplitude is preceded by inversion of the perturbation phase. This effect was first revealed experimentally [2] and then it was confirmed theoretically by numerical solution of the equations of hydrodynamics [3].

With passage of the shock, the interface between two media acquires impulsive acceleration, i.e., the acceleration time is equal to zero and the increase in velocity is finite. The flow produced by constant (in time) acceleration of the interface has been adequately studied. It is known that for $\rho_1 > \rho_2$, this flow is stable [in contrast to the case $\rho_1 < \rho_2$, where the well-known Rayleigh–Taylor instability occurs with perturbation amplitude increasing exponentially with time]. The motion of the interface with acceleration that is constant in time can be approximately replaced by a sequence of impulsive accelerations following one after another at a short time interval which goes to zero in the limit. For $\rho_1 > \rho_2$, this implies the possibility of obtaining steady flow from a sequence of unsteady flows, and this is related to the phase inversion described above. Indeed, the perturbation amplitude first decreases when phase inversion occurs, and its subsequent growth is prevented by the next impulsive acceleration which again results in phase inversion.

The question arises of whether it is possible to weaken the Richtmyer–Meshkov instability for $\rho_1 > \rho_2$ by a second shock propagating in the same direction as the first. Apparently, the development of the instability

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in the case of two successive shocks was first considered in [9] for potential flow of incompressible liquids with impulsive acceleration of the interface. Below, we present a nonrigorous derivation of the formula for the growth rate of the amplitude of a sinusoidal perturbation, based on physical arguments and Richtmyer's formula for a single shock:

$$\dot{a} = \frac{2\pi a_0}{\lambda}\,\mathrm{A}v, \qquad \frac{2\pi|a_0|}{\lambda} \ll 1, \tag{1}$$

where $a = a(t)$ is the perturbation amplitude as a function of time t, a_0 is the initial amplitude, λ is the perturbation wavelength, $\mathrm{A} = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ is the Atwood number, and v is the change in the velocity of the interface after passage of the shock; a dot denotes differentiation with respect to time.

Strictly speaking, formula (1) corresponds to impulsive acceleration of the interface between two incompressible liquids rather than to passage of a shock through the interface. In the case of a shock, Richtmyer's formula contains the densities ρ_1 and ρ_2 and the amplitude a_0 immediately after passage of the shock rather than the initial values of these quantities. For the subsequent consideration, which is qualitative in character, this remark is however not important.

Richtmyer obtained formula (1) for the case $\rho_1 < \rho_2$ ($\mathrm{A}v > 0$). However, later it became clear that it also applies for the case $\rho_1 > \rho_2$ ($\mathrm{A}v < 0$) if the change of sign of the amplitude $a(t)$ is treated as phase inversion.

After passage of the first shock,

$$\dot{a} = \dot{a}_1 = \frac{2\pi}{\lambda}\,\mathrm{A}a_0 v_1, \tag{2}$$

where v_1 is the velocity of the interface after the first shock. Since we consider here the case $\mathrm{A} < 0$, $v_1 > 0$, it is convenient to set $a_0 < 0$. Then, $\dot{a}_1 > 0$ and, after inversion of the phase of the initial perturbation, the amplitude $a > 0$.

Let the interface be instantaneously accelerated to velocity $v_1 + \Delta v$ after passage of the second shock. Using Richtmyer's formula again, for the amplitude growth rate after passage of the second shock, we obtain

$$\dot{a} = \dot{a}_2 = \dot{a}_1 + \beta_m \mathrm{A}\,\Delta v, \qquad \beta_m = \frac{2\pi a_m}{\lambda}, \tag{3}$$

where a_m is the perturbation amplitude at the moment t_2 of arrival of the second shock and β_m is the relative amplitude. The amplitude a_m can be expressed in terms of t_2 using (2):

$$\dot{a}_2 = \dot{a}_1\Big(1 + \frac{\Delta v}{v_1} + \frac{2\pi}{\lambda}\,\mathrm{A}\Delta v t_2\Big). \tag{4}$$

For $\mathrm{A} < 0$, $\lambda > 0$ exists such that $\dot{a}_2 = 0$. This effect is called freezing. In general, however, formula (4) gives an unfavorable prediction for the development of the instability at large times. Indeed, for specified Δv and t_2 and for $\lambda \to 0$, we have $\dot{a}_2/\dot{a}_1 \to -\infty$, i.e., the second shock can indefinitely increase the growth rate of a short-wave perturbation. It should be taken into account, however, that the conclusion drawn above is valid only if at the moment of arrival of the second shock, the instability of the interface is at the linear stage ($\beta_m \ll 1$). Formally, this follows from the applicability condition for Richtmyer's formula.

For the nonlinear evolution of perturbations ($\beta_m \sim 1$), a theoretical study of the effect of additional acceleration of the interface is possible only by numerical calculations of the equation of hydrodynamics. For example, a numerical study [10] of the stability of shaped-charge jets in conical targets revealed the following mechanism of weakening of the Richtmyer–Meshkov instability. A leading shock passing from aluminum into deuterium initiated instability of the interface. In addition to this, one more compression wave moved along the interface, and the corresponding increase in pressure past the front of this wave resulted in an additional acceleration of the interface. The phase inversion of the perturbation due to this acceleration and the associated drop in the amplitude occurred even when the evolution of the instability was substantially nonlinear.

There has been just a few experimental studies of the stability of the interface for two successive shocks. In some experiments with shock tubes, besides the initial shock, secondary shock waves, reflected from the

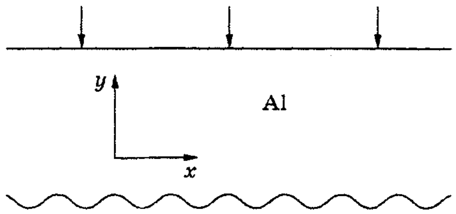

Fig. 1

tube walls, also affected the interface (see, e.g., [11]). However, the pure case of a sequence of two shocks propagating in the same direction has not been studied. In these experiments, at least one of the secondary waves was opposite in direction to the initial wave. As far as we are aware, the only experimental study of two shocks going in the same direction in the case $A < 0$ was performed by Dimonte et al. [8]. In the experiment of [8], the initial shock was generated by strong x-radiation, and the second shock appeared after reflection of the depression wave from the ablation front. The wavelength of the interface perturbation varied over a wide range. As follows from (2), the growth rate of the amplitude of a long-wave perturbation is comparatively small. At the moment of arrival of the second shock wave, the amplitude of the long-wave perturbation was so small that the second wave practically did not affect the development of the instability. For a short-wave perturbation, the situation was different. By the moment of arrival of the second shock, the perturbation amplitude was sufficiently large, so that this wave sharply slowed down the further growth in amplitude. As a result, as shown in [8], at the moment of measurement, the amplitude of the short-wave perturbation was even smaller than that of the long-wave perturbation.

In the present paper, we study numerically Richtmyer–Meshkov instability for two successive shocks in the case $A < 0$. In contrast to [9], here primary attention is given to values $\beta_m \sim 1$.

Formulation of the Problem and Numerical Method. We consider the instability of a free surface of aluminum after passage of two successive shocks that follow one after the other at a certain time interval. The first shock is rather strong, so that we can use the equations of hydrodynamics. Viscosity and thermal conductivity are ignored. For the chosen parameters, the velocity past the first shock is 3 km/sec, the pressure is 75 GPa, and the aluminum is in the liquid state. The equations of state for aluminum had the form of tables of values of the pressure and internal energy versus the temperature and density, compiled in accordance with [12].

Let us define the problem more accurately. We consider two-dimensional flow that depends on two Cartesian coordinates x and y and is unbounded along the x axis (see Fig. 1). At the time $t = 0$, the aluminum layer is at atmospheric pressure and room temperature. On the upper boundary of the layer, the velocity of the substance is prescribed:

$$u_x = 0, \qquad u_y = \begin{cases} -u_1, & 0 \leqslant t \leqslant \tau, \\ -(u_1 + \Delta u), & t > \tau \end{cases}$$

(τ is the delay of the second shock). For $\Delta u > 0$, this boundary condition initiates two successive shocks in aluminum at $t = 0$ and τ. For specified thickness of the layer h and velocity $u_1 > 0$, the parameters Δu and τ are chosen so that the first shock enters the lower boundary of the layer before being overtaken by the second shock. The lower boundary is at atmospheric pressure. The velocity of the lower boundary after arrival of the first and second shocks at it was determined by numerical calculation of the corresponding one-dimensional problem using rather fine meshes. For a velocity $u_1 = 3$ km/sec, the velocity of the lower boundary $v_1 \approx 6.2$ km/sec, and this agrees with the rule of doubling of velocities [13] for weak shocks. This rule is also satisfied for the second shock, whose arrival at the lower boundary increases the velocity of the boundary by $\Delta v \approx 2\Delta u$. The layer thickness $h = 5$ mm is chosen such that the perturbation of the lower boundary does not reach the upper boundary in the time of numerical calculation. At the moment $t = 0$, the upper boundary has coordinate $y = h$, and the equation of the lower boundary is

$$y(x) = \frac{\varepsilon}{2} \cos \frac{2\pi x}{\lambda},$$

where ε and λ are the amplitude and wavelength of the initial perturbation. It is sufficient to solve the problem in the interval $0 \leqslant x \leqslant \lambda/2$ with a symmetry condition at its ends specified in such a manner that in the absence of dissipation, it is equivalent to the condition on a rigid wall.

The numerical calculations were performed using the same software as in [10]. All the boundaries of the domain including the free surface were chosen explicitly as certain lines of a regular curvilinear mesh. We used second-order quasimonotonic schemes based on Godunov's scheme. The only difference from [10] was a new method of calculating curvilinear meshes: instead of the method of [14] we used its modification proposed in [15], which improved the quality of the mesh near the free surface while preserving the convexity of all quadrangular cells of the mesh.

Along the upper boundary and the free surface, the mesh points were placed uniformly. Along the side boundaries $x = 0$ and $x = \lambda/2$, the distribution of mesh points was nonuniform since values $\lambda \ll h$ were taken. Near the free surface, the mesh points were located uniformly with a mesh step of the same order as that along the free surface. Away from the free surface, the mesh step along the side boundaries was gradually increased, so that the ratio of the mesh steps for adjacent cells did not exceed 1.05. Apart from the calculations using a two-dimensional scheme for the entire domain, we carried out calculations with an artificial internal boundary $y = y_*(t)$, above which for $y > y_*$ the flow was considered one-dimensional and was calculated using a one-dimensional second-order finite-difference scheme in Lagrangian variables. Near the artificial boundary, the step of the one-dimensional Lagrangian mesh coincided with the step of the two-dimensional mesh along the side boundaries with an accuracy of a few percent. The two-dimensional mesh had 50–100 intervals along the free surface. Along the side boundaries, the mesh had 80–160 intervals in calculations with the artificial internal boundary and 200–300 in calculations without it. Test calculations were carried out using various meshes for the same values of the parameters of the problem. These calculations reproduced each other fairly well.

In the numerical method used here, it is necessary to reconstruct the mesh if the simple connectedness of the flow domain is broken, and this is a time-consuming procedure. Therefore, if for the specified parameters of the problem, the simple connectedness of the domain is broken at some time, the calculation is carried out only up to this moment.

Results of Calculations. In what follows, the results will be compared with formula (3). For the free-surface problem considered here, one should set $\rho_2 = 0$ and, hence, $\mathrm{A} = -1$. The change in the free-surface velocity Δv after passage of the second shock is determined in accordance with the above-mentioned rule of doubling of velocities $\Delta v = 2\Delta u$. The remaining quantities in (3) are obtained using the function $\Delta y(t)$, which is the difference between the maximum and minimum values of the y coordinate of the boundary. The difference derivative of the function $\Delta y(t)$ has small oscillations, which are filtered out by averaging over the period of oscillations. We denote the result of the averaging by $\Delta\dot{y}(t)$.

To determine the moment t_2 when the second shock comes to the free-boundary, we compare the function $\Delta y(t)$ with an analogous function obtained in a calculation without a second shock. The time from which these functions become different is taken as t_2. As a result, we obtain the growth rate of the amplitude before passage of the second wave $\dot{a}_1 = \Delta\dot{y}(t_2)$ and the amplitude $a_m = \Delta y(t_2)$, which, in turn, determines the parameter β_m in (3). Then, for the growth rate of the amplitude after passage of the second wave, we set $|\dot{a}_2| = |\Delta\dot{y}(t_2')|$ for some $t_2' > t_2$. The choice of t_2' and the sign of $\dot{a}_2$ depend on the form of the function $\Delta y(t)$ in a particular calculation. If the sign of $\Delta\dot{y}(t)$ changes at some $t_2^0 > t_2$, negative $\dot{a}_2$ are assumed and the maximum value of $|\Delta\dot{y}(t)|$ for $t > t_2^0$ is chosen as a $|\dot{a}_2|$. If the sign of $\Delta\dot{y}(t)$ does not change for $t > t_2$ then $\dot{a}_2$ is assumed to be positive and the inflection point of the function $\Delta y(t)$ or the maximum time of calculation are chosen as t_2'.

Thus, from the calculation result we independently determine the parameter β_m and the change in the amplitude growth rate $\Delta\dot{a} = \dot{a}_1 - \dot{a}_2$. Instead of the latter, it is convenient to introduce the parameter β^*:

$$\Delta\dot{a} = -\beta^* \mathrm{A} \Delta v. \tag{5}$$

TABLE 1

λ	β_m	$\dot{a}_1$	$-\dot{a}_2$	β^*	$\dot{a}_1^{RBM}$
0.2	1.8	0.54	0.8	1.34	0.51
0.4	0.46	0.29	0.14	0.43	0.26
1.0	0.075	0.11	−0.035	0.075	0.11

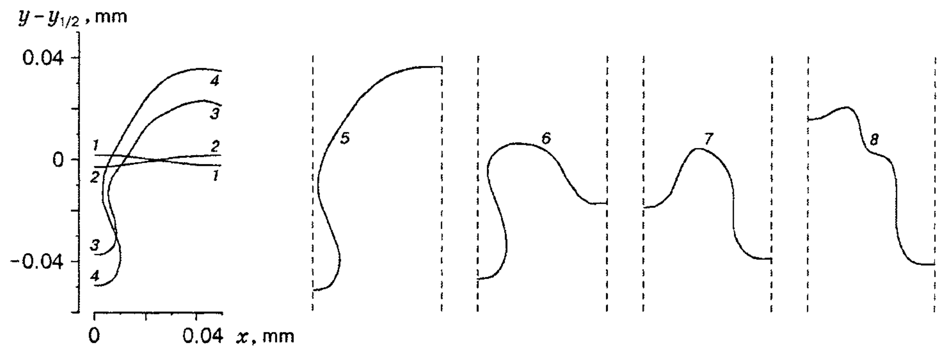

Fig. 2

It is easy to see that (5) coincides with (3) if $\beta^* = \beta_m$.

We first consider results for small values of β_m for which Eq. (3) is valid. For fixed parameters of the shocks $u_1 = 3$ km/sec, $\Delta u = 0.5$ km/sec, and $\tau = 0.27$ μsec and an amplitude of the initial perturbation $\varepsilon = 0.004$ mm, we consider wavelengths $\lambda = 0.2$, 0.4, and 1.0 mm. The calculation results are shown in Table 1 ($\dot{a}_1^{RBM}$ is the growth rate of the perturbation amplitude obtained using the model of [3]). The parameter β_m is inversely proportional to λ^2 and decreases rapidly with increase in λ. Obviously, the smaller the parameter β_m, the less it differs from β^*. Thus, the calculations agree with (3) in the range of validity of this equation.

Immediately after passage of the first shock, the initial amplitude of the perturbation decreases somewhat. This decreased amplitude a'_0 can be determined approximately by analyzing the numerical results. Table 1 (column 6) shows the values of the derivative $\dot{a}_1$ after passage of the first shock, calculated using formula (2), where, the initial amplitude a_0 is replaced by the quantity $(a_0 + a'_0)/2$. Meyer and Blewett [3] proposed this model for the case A < 0, analyzing results of calculations using the equations of hydrodynamics simulating experiments with gases in shock tubes. Dimonte et al. [8] note that the model is in satisfactory agreement with an experiment in which both media were in the solid state before arrival of the shock wave. As follows from Table 1, the values of $\dot{a}_1$ obtained differ from the model of [3] by not more than 10%.

Let us analyze the main results obtained for large values of β_m. Figure 1 shows the free-boundary shape for several successive times $t = 0$, 0.54, 0.61, 0.65, 0.67, 0.75, 0.81, and 0.85 μsec (curves 1–8). The shock-wave parameters u_1, Δu, and τ have the same values as in the previous calculations. The parameters of the initial perturbation are $\varepsilon = 0.004$ mm and $\lambda = 0.1$ mm. Along the ordinate axis, we plot the deviation of the y coordinate of the boundary from a certain mean value $y_{1/2}$, which is defined for each position of the boundary as the y coordinate of the point located in the middle of the boundary. The initial shape of the boundary (curve 1) has a maximum of the y coordinate at $x = 0$ and a minimum at $x = \lambda/2$. The remaining curves correspond to times after passage of the first shock. One can see that the perturbation phase changed: the y coordinate now has the minimum at $x = 0$ and the maximum at $x = \lambda/2$.

Curves 2–4 in Fig. 2 show the process of formation of an aluminum drop near $x = 0$. In the absence of a second shock wave, this drop would separate from the main body of aluminum. Formation of the first drop can be regarded as the beginning of formation of a sheet of many drops, which is similar to the stage of turbulent mixing at the interface between two media.

In the above example, the parameters of the problem are selected so that at the moment of arrival of

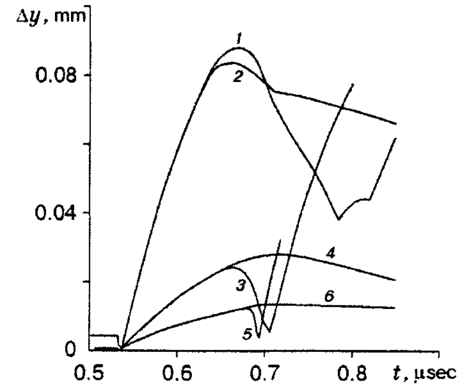

Fig. 3

TABLE 2

λ	ε	τ	β_m	$\dot{a}_1$	$-\dot{a}_2$	β^*
0.1	$4 \cdot 10^{-3}$	0.19	1.0	1.0	0	1.0
0.1	$4 \cdot 10^{-3}$	0.20	2.0	1.0	0.5	1.5
0.1	$4 \cdot 10^{-3}$	0.22	3.5	0.76	0.62	1.38
0.1	$4 \cdot 10^{-3}$	0.27	5.5	0.65	0.60	1.25
0.05	$5 \cdot 10^{-4}$	0.19	0.57	0.25	0.28	0.53
0.05	$5 \cdot 10^{-4}$	0.20	0.94	0.25	0.55	0.80
0.05	$5 \cdot 10^{-4}$	0.22	1.7	0.22	0.92	1.14
0.05	$5 \cdot 10^{-4}$	0.27	3.0	0.145	1.05	1.2
0.05	$5 \cdot 10^{-4}$	0.32	4.0	0.1	0.97	1.07
0.02	$8 \cdot 10^{-5}$	0.27	3.9	0.07	1.35	1.4

the second wave, the small bridge separating the drop from the main body of aluminum still exists. Curves 5–8 (Fig. 2) show the evolution of the free boundary after passage of the second shock. Being almost detached (see curve 5), the drop is gradually absorbed by the main body of aluminum. At the same time, an aluminum "tongue" forms and begins to grow near the opposite boundary $x = \lambda/2$. Qualitatively, this process does not differ from the process of phase inversion due to passage of the first shock wave.

The amplitude of oscillations of the boundary $\Delta y = y_{\max} - y_{\min}$ versus time is shown in Fig. 3. After arrival of the first shock at $t \approx 0.53$ μsec, the phase of the initial perturbation changes, and the amplitude decreases almost to zero (curve 1; $\lambda = 0.1$ mm and $\Delta u = 0.5$ km/sec). Before arrival of the second shock at $t \approx 0.65$ μsec, the amplitude increases. After that, phase inversion occurs again and the amplitude decreases. In Fig. 3, the inflection points of curve 1 correspond to the moments when boundary points with extremum values of y changes their position instantaneously. The increase in the amplitude at the end of calculation is related to the growing aluminum "tongue" near the boundary $x = \lambda/2$.

As τ increases, the time of arrival of the second shock wave increases. Starting from a certain value τ_0, the aluminum drop is completely separated from the main body, and the simple connectedness of the flow domain is broken. As mentioned above, to perform a calculation for this case using the proposed numerical method, a time-consuming procedure is needed for changing the mesh structure. Such calculations have not been performed. However, one should take into account that a slight increase in τ does not lead to qualitative changes in the time dependence of the amplitude. Additional acceleration of the main body of aluminum due to arrival of the second shock wave will result in a rapid attachment of the separated drop to the main body.

Results of the main series of calculations for large values of β_m are given in Table 2. In contrast to Table 1, Table 2 gives calculation results for perturbations with shorter wavelengths $\lambda = 0.10$, 0.05, and 0.02 mm for $u_1 = 3$ km/sec and $\Delta u = 0.5$ km/sec. The initial amplitude ε for each λ was chosen so that in the case of arrival of only the first shock wave, the relative amplitude $\beta = 2\pi\Delta y/\lambda$ reached sufficiently large values at the same time for all cases. For perturbations with $\lambda = 0.10$ and 0.05 mm, we carried out calculations for various delays of the second shock τ. An increase in τ leads to an increase in the relative amplitude β_m at the moment of arrival of the second shock.

As β_m increases, the parameter β^* in formula (5) first grows and then decreases. This means that at the nonlinear stage of instability development, the change in the amplitude growth rate $\Delta\dot{a}$ ceases to depend significantly on the relative amplitude β_m by the moment of arrival of the second shock wave. From a comparison of the values of β^* obtained for various values of λ, one more important conclusion follows: at the nonlinear stage of instability development, the quantity $\Delta\dot{a}$ depends weakly on the perturbation wavelength. Thus, assigning a certain mean value of β^* in formula (5), we can choose the change in the velocity of the boundary after arrival of the second shock Δv if only the amplitude growth rate $\dot{a}_1$ is known. In particular, setting $\dot{a}_2 = 0$ in (5) we obtain the freezing regime. In the boundary condition, the parameter of the second shock is $\Delta u = \Delta v/2$ according to the rule of velocity doubling. For strong shocks, Δu can be determined from a series of one-dimensional calculations. To validate this approach, we performed the following test calculations. In Table 2, the values of the parameter β^* for wavelengths $\lambda = 0.10$, 0.05, and 0.02 mm and the maximum values of τ are equal to 1.25, 1.07, and 1.40, respectively. In all test calculations, we used the mean value $\beta^* = 1.25$. Then, for the specified wavelengths we take the corresponding values of $\dot{a}_1$ from Table 2 and, using formula (5), determine the values of

$$\Delta v = -\frac{\dot{a}_1}{A\beta^*}, \qquad \Delta u = \frac{\Delta v}{2}. \tag{6}$$

Figure 3 shows curves of the perturbation amplitude versus time for $\Delta u = 0.5$ km/sec (curves 1, 3, and 5) and Δu obtained from formula (6) (curves 2, 4, and 6), for $\lambda = 0.10$, 0.05, and 0.02 mm (curves 1 and 2, 3 and 4, and 5 and 6, respectively). For $\lambda = 0.1$ mm, the parameter τ was slightly decreased to avoid separation of the drop. It is obvious that for the three values of λ considered, the desired effect was achieved.

Let us describe calculation results for the case where the initial perturbation is not sinusoidal with a certain wavelength. This problem is also solved in the interval $0 \leqslant x \leqslant \lambda/2$ with the above symmetry condition at its ends, and the initial perturbation is specified by a broken line with three segments:

$$y(x) = \frac{\varepsilon}{2}\begin{cases} 1 - 2x/(0.2\lambda), & 0 \leqslant x \leqslant 0.2\lambda, \\ -1 + 2(x - 0.2\lambda)/(0.1\lambda), & 0.2\lambda \leqslant x \leqslant 0.3\lambda, \\ 1 - 2(x - 0.3\lambda)/(0.2\lambda), & 0.3\lambda \leqslant x \leqslant 0.5\lambda, \end{cases}$$

where the central segment is half the end segments. In Fig. 4, the evolution of this perturbation is shown for $\lambda = 0.2$ mm and $\varepsilon = 0.002$ mm at $t = 0$, 0.55, 0.60, and 0.66 μsec (curves 1–4) after a shock with $u_1 = 3$ km/sec arrives at the free surface. One can see that phase inversion occurs, the boundary rapidly takes a complicated shape, and a drop is formed in the neighborhood of the point $x = 0.3\lambda = 0.06$ mm, which is an extremum point of the initial perturbation.

To find the parameters of the second shock, we use the method described above. From the calculation results, we obtain the value of $\dot{a}_1$ at a certain time. Formulas (6) with $\beta^* = 1.25$ are used to calculate the parameter Δu. Then, we perform a series of one-dimensional calculations to determine the delay τ of the second shock that ensures its arrival at the free surface at the required time. Figure 5 shows results of this calculation (curve 2) and also results of a calculation without a second shock (curve 1). One can see that the freezing effect is achieved.

Conclusion. For shock waves in aluminum with a postshock pressure of about 75 GPa, we obtained the following result. If at the moment the second shock arrives at the free boundary, the perturbation evolution is at the nonlinear stage, then, in contrast to the linear stage, the change in the amplitude growth rate depends weakly on the wavelength of the initial perturbation. This makes it possible to determine parameters of the second shock that ensure freezing of the instability using the value of the amplitude growth rate, obtained, for example, from an experiment.

The proposed method for obtaining parameters of the second shock is based on formula (5), in which the structure of Richtmyer's formula is preserved. Thus, this method may be used for different shock parameters

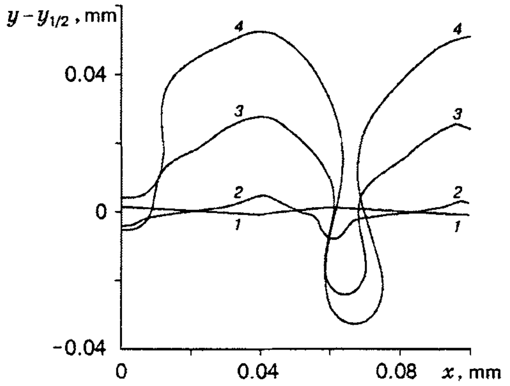

Fig. 4

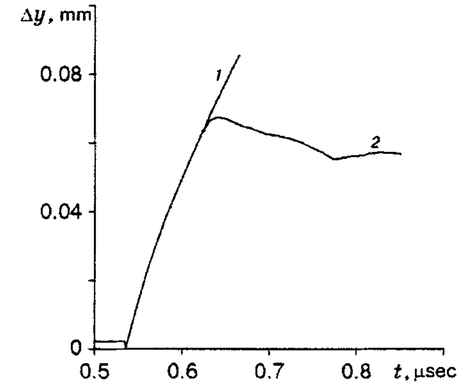

Fig. 5

and for different media, including flow with an interface between two media. Conceivably, the proposed value of the parameter $\beta^* = 1.25$ will not require significant correction.

There is still an open question of the choice of parameters of the second shock if at the moment it enters the boundary, the evolution of perturbations is at the stage of turbulent mixing. For example, in experimental studies of Rayleigh–Taylor instability [16, 17], acceleration is first directed from a light material to a heavy material, and this leads to rapid development of turbulence at the interface. Then, the direction of the acceleration changes. As a result, a separation phenomenon is observed, i.e., the width of the turbulent mixing region decreases. It is of interest to perform the corresponding experiments with shocks. It is not impossible that even at the stage of turbulent mixing, the impact of the second shock is approximately described by formula (5) with the appropriate choice of the parameter β^*.

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REFERENCES

1. R. D. Richtmyer, "Taylor instability in shock acceleration of compressible fluids," *Comm. Pure Appl. Math.*, **13**, No. 2, 297–319 (1960).
2. E. E. Meshkov, "Instability of an interface between two gases accelerated by a shock," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 5, 151–157 (1969).
3. K. A. Meyer and P. J. Blewett, "Numerical investigation of the stability of a shock-accelerated interface between two fluids," *Phys. Fluids*, **15**, No. 5, 753–759 (1972).
4. N. N. Anuchina, S. M. Bakhrakh, A. V. Zabrodin, et al., "Hydrodynamic instability of an interface between two media," in: *Investigation of Hydrodynamic Stability Using Computers* [in Russian], Inst. of Appl. Math., USSR Acad. of Sci., Moscow (1981), pp. 108–162.
5. A. N. Aleshin, V. E. Lazareva, S. G. Zaitsev, et al., "Linear, nonlinear, and intermediate stages of development of Richtmyer–Meshkov instability," *Dokl. Akad. Nauk SSSR*, **310**, No. 5, 1105–1108 (1990).
6. V. E. Neuvazhaev, "Development of turbulent mixing due to Richtmyer–Meshkov instability," *Mat. Model.*, **3**, No. 7, 10–28 (1991).
7. I. G. Lebo, V. V. Nikishin, V. B. Rozanov, and V. F. Tishkin, "On the influence of boundary conditions on the development of contact surface instability due to arrival of a shock," *Krat. Soobshch. Fiz.*, Nos. 1/2, 48–57 (1997).
8. G. Dimonte, C. E. Frerking, M. Schneider, and B. Remington, "Richtmyer–Meshkov instability with strong radiatively driven shocks," *Phys. Plasmas*, **3**, No. 2, 614–630 (1996).

9. K. O. Mekaélian, "Richtmyer–Meshkov instabilities in stratified fluids," *Phys. Rev. A*, **31**, No. 1, 410–419 (1985).
10. A. A. Charakhch'an, "Stability of shaped-charge jets generated by pulsed action on conical targets," *Prikl. Mekh. Tekh. Fiz.*, **38**, No. 3, 10–13 (1997).
11. V. E. Meshkov, "Results of experimental studies of gravitational instability on an interface between media with different densities," in: *Investigation of Hydrodynamical Stability Using Computers* [in Russian], Inst. of Appl. Math., USSR Acad. of Sci., Moscow (1981), pp. 162–190.
12. A. V. Bushman, G. I. Kanel', A. L. Ni, and V. E. Fortov, *Thermophysics and Dynamics of Intense Impulsive Impacts* [in Russian], Inst. for Chem. Phys., Chernogolovka (1988).
13. Ya. B. Zel'dovich and Yu. P. Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena*, Academic Press, New York (1967).
14. S. A. Ivanenko and A. A. Charakhch'an, "Curvilinear meshes with convex quadrangular cells," *Zh. Vychisl. Mat. Mat. Fiz.*, **28**, No. 4, 503–514 (1988).
15. A. A. Charakhch'an, "Elliptic generator of meshes based on quasi-one-dimensional meshes," *Zh. Vychisl. Mat. Mat. Fiz.*, **39**, No. 5, 832–837 (1999).
16. D. L. Youngs, "Modeling turbulent mixing by Rayleigh—Taylor instabilities," *Physica D*, **37**, Nos. 1/3, 270–287 (1989).
17. Yu. A. Kucherenko, V. E. Neuvazhaev, and A. P. Pylaev, "Behavior of a gravitational mixing region under conditions leading to separation," *Dokl. Ross. Akad. Nauk*, **334**, No. 4, 445–448 (1994).